



Exponentiation:

Basic: $a^m \cdot a^n = a^{m+n}$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

etc.

Factorial:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots (2)(1)$$

Functions:

General

- **Definition**
 - assign each $x \in A$ to exactly one $y \in B$ where, $f: A \rightarrow B$
 - i.e. each x only has one y

- **Characteristics**

$$f: A \rightarrow B$$

- set of $A = \text{domain} = \text{dom}(f)$

- Set of $B = \text{range / target / codomain} = \text{rg}(f)$

- **Sets**

- set-builder:

$$\{x \mid 1 \leq x < 9\}$$

- interval:

$$[1, 9)$$

[: included

(: not included

- **Operations**

- $(f+g)(x) = f(x) + g(x)$

- $(f-g)(x) = f(x) - g(x)$

- $(fg)(x) = f(x)g(x)$

- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Types

• Composition of Functions

- $(h \circ g)(x) = h(g(x))$ (not commutative, i.e. $h \circ g \neq g \circ h$)
- Notation: f^{-1} is the inverse of f
 - f must be one-to-one as f^{-1} is a function where each x only has $\underline{1}$ y (as the y 's and x 's swap)
- Symmetric across $y=x$
- f^{-1} gives an x for an assigned y whereas f gives a y for an assigned x
If f has point (a, b) , f^{-1} will have (b, a)
- Hence, $f^{-1}(f(x)) = x$
Thus, to find f^{-1} : if $f(x) = x^2 + x$
 $y = x^2 + x$
then solve:
 $x = \frac{y^2 + y}{2}$
by switching x and y

• Polynomial Function

- Form: $f(x) = \sum_{k=0}^n a_k x^k$, $a_k \in \mathbb{R}$
and obviously $n \geq 0$, $n \in \mathbb{Z}^+$
- Each term/monomial is thus a power term
- Integral: $\sum_{k=0}^n \frac{a_{k+1}}{k+1} x^{k+1} + C$
or
 $\sum_{k=0}^n \frac{a_k}{k} x^{k+1} + C$

• Odd and even

Definitions

- odd: $f(-x) = -f(x)$
(only odd exponents)
- even: $g(x) = g(-x)$
(only even exponents)

Operations

$f_1(x)$: odd, $g_1(x)$: even

Addition/Subtraction

- $f(x) \pm g(x)$ is neither (as will have both even and odd exponents)
- $f_1(x) + f_2(x)$ is odd
- $g_1(x) + g_2(x)$ is even

Multiplication/Division

- $\frac{f(x)}{g(x)}$ is odd
- $\frac{f_1(x)}{f_2(x)}$ is even
- $\frac{g_1(x)}{g_2(x)}$ is even

Solving technique

$$\begin{aligned}\frac{f(x)}{g(x)} &= h(x) \\ \Leftrightarrow \frac{f(-x)}{g(-x)} &= h(-x) \\ \Leftrightarrow \frac{-f(x)}{g(x)} &= h(-x) \quad (\text{by } f \text{ odd, } g \text{ even}) \\ \Rightarrow -h(x) &= h(-x) \quad \therefore h(x) \text{ is odd}\end{aligned}$$

Composites

$f(g(x))$: even
 $f_1(g_2(x))$: odd
 $g_1(g_2(x))$: even

$$\left. \begin{aligned}f(g(x)) &= h(x) \\ \Leftrightarrow f(g(-x)) &= h(-x) \\ \Leftrightarrow f(g_2(x)) &= h(-x) \\ \therefore h(x) &\text{ is even}\end{aligned} \right\}$$

• e^x

• As a polynomial: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

• $\ln(x)$ is the inverse of e^x

thus, $\ln(e^x) = x$

$\Rightarrow e^{\ln(x)} = x$ $\begin{cases} \ln(x) = m \\ \Rightarrow \ln(e^{\ln(x)}) = \ln(x) \\ \Rightarrow \ln(x) = \ln(m) \therefore x = m \end{cases}$ NB: bring neg down using ln on RHS and LHS

• Trig Functions

• Basic

Function	Definition	Domain	Set of principal values
$y = \arcsin x$	$\sin y = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x$	$\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x$	$\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arccot } x$	$\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \text{arcsec } x$	$\sec y = x$	$x < -1$ or $x > 1$	$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$
$y = \text{arcsc } x$	$\csc y = x$	$x < -1$ or $x > 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$

• Calculus

$f(x)$	$f'(x)$	$\int f(x) dx$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln \sec x + C$
$\csc x$	$-\csc x \cot x$	$\ln \csc x - \cot x + C$
$\sec x$	$\sec x \tan x$	$\ln \sec x + \tan x + C$
$\cot x$	$-\csc^2 x$	$-\ln \csc x + C$

• Special angles

Angle, θ , in		$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
radians	degrees			
0	0°	0	1	0
$\frac{\pi}{12}$	15°	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$2 - \sqrt{3}$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{5\pi}{12}$	75°	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$2 + \sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	Undefined

• Complex

• $e^{ix} = \cos x + i \sin x$

• $\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$

• $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

{ shown through Maclaurin of e^{ix} , $\sin(x)$ and $\cos(x)$ }

Good to know

• $\cos^2 x + \sin^2 x = 1$

• $\sin(2x) = 2 \sin(x) \cos(x)$

• $\cos(2x) = \cos^2(x) - \sin^2(x)$
 $= 2\cos^2(x) - 1$
 $= 1 - 2\sin^2(x)$

• $1 + \tan^2 x = \sec^2 x$

• $1 + \cot^2 x = \csc^2 x$

• $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

• $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

• $\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a)\tan(b)}$

• $\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$

• $\frac{d}{dx} \arccos(x) = \frac{-1}{\sqrt{1-x^2}}$

• $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$

• Hyperbolic

Basic

- $\cosh(x) = \frac{e^x + e^{-x}}{2}$
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$
- $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \left\{ \tanh(x) = \frac{\sinh(x)}{\cosh(x)}, \text{ just by definition} \right\}$

Inverse

- $\operatorname{sech}(x) = \frac{1}{\cosh(x)}$
 - $\cosh(x) = \frac{1}{\operatorname{sech}(x)}$
 - $\operatorname{coth}(x) = \frac{1}{\tanh(x)}$
- { similarly, just by definition }

Additional

- $\cosh^2(x) - \sinh^2(x) = 1$
- $\cosh(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$
- $\sinh(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$

Explanation

- The similarity of $\cosh$ and $\sinh$ to $\cos$ and $\sin$, is that equations of hyperbola: $x^2 - y^2 = 1$
of circle: $x^2 + y^2 = 1$

And trig. functions based on the unit circle.

